The quantum mechanical Aharonov-Bohm effect is generally seen as forcing us to revise our interpretation of the classical theory of electromagnetism. This is somewhat puzzling: How can a feature of false theory mandate a revision of an interpretation of another false theory? I attempt to make sense of this phenomenon by arguing that the various possible interpretations of classical electromagnetic theory underwrite distinct approaches to quantization. I infer that our interpretative beliefs about electromagnetism are open to revision in light of quantum effects. I sketch an approach to understanding the structure and content of our physical knowledge which makes sense of this inference.

1. Introduction

When one first learns classical electromagnetism, one is taught to think of Maxwell’s equations as governing the evolution in time of the electric and magnetic fields (or, more subtly, of the electromagnetic field). Under this interpretation the theory is both deterministic and local. Here, ‘deterministic’ means that, according to the theory, specifying the present state of the fields suffices to fix their future and past histories. ‘Local’ means that if we want to know what will happen next here, the theory tells us that we need only look at the field values hereabouts right now—we need not know what is happening arbitrarily far away. Thus construed, electromagnetism is the paradigm of all that a classical (i.e. non-quantum) theory should be.
Although this way of thinking about electromagnetism remains the pedagogical standard, it has been known for some time to be untenable. In 1959, Aharonov and Bohm argued that a charged quantum particle moving in the region external to a solenoid would be sensitive to whether or not there was current running through the device, despite the fact that the field values in the regions of space occupied by the particle would be unaffected by the operation of the solenoid. This effect was subsequently detected in experiments involving beams of electrons.

It is widely agreed that the experimental confirmation of the Aharonov-Bohm effect discredits the familiar way of understanding electromagnetism. One can maintain the traditional interpretation of the theory only at the price of admitting that the fields act where they are not. But this position flies in the face of the well-entrenched principle that classical fields act by contact rather than at a distance. It would seem, then, that the electric and magnetic fields can’t constitute the ontology of electromagnetism. It is now standard to maintain the Aharonov-Bohm effect shows that the vector potential, formerly viewed as a mere mathematical convenience, must in fact be physically real. The advantage here is that the vector potential in the region exterior to the solenoid, unlike the fields, depends on whether or not the device is operational, so that one can explain the behavior of the particle in terms of the values of vector potential in the region actually occupied by the particle.

Thus, it is often said that the Aharonov-Bohm effect shows that the traditional interpretation of electromagnetism must be replaced. I subscribe to this conclusion. But I would put it somewhat differently: until the discovery of the Aharonov-Bohm effect, we misunderstood what electromagnetism was telling us about our world. This formulation captures what I take to be the kernel of the common wisdom. But it is intentionally provocative: it brings to the fore the epistemological and metaphysical the puzzles inherent in episodes like the post-Aharonov-Bohm reinterpretation of electromagnetism.
After all, by the time the Aharonov-Bohm effect was discovered, it had long since been accepted that electromagnetism does not accurately describe our world. In a widely influential paper of 1933, Bohr and Rosenfeld argued that there can be no consistent theory of the interaction between charged quantum particles and a classical electromagnetic field.\(^1\) Thus our world could not possibly contain the sort of field described by Maxwell’s equations: electromagnetism is a false theory. Now, there is a very straightforward sense in which a false—but eminently useful—theory like electromagnetism can tell us about our world: it makes empirical predictions which are very accurate within certain circumscribed domains of applicability. But it seems strange to say that the interpretation of such a theory tells us about our world. To interpret a theory is to describe the possible worlds about which it is the literal truth. Thus, an interpretation of electromagnetism seems to tell us about a possible world which we know to be distinct from our own. On the other hand, whatever world electromagnetism is true of, it is not one which contains quantum electrons. So it is difficult to see how a quantum mechanical effect can teach us anything about the interpretation of electromagnetism. Of course, quantum mechanics itself is false (being nonrelativistic). So our world is one about which neither electromagnetism nor quantum mechanics is true. Nonetheless, I maintain, we learn something about our own world when we study the interpretative interaction between these two false theories.

In this paper I present a logical reconstruction of the Aharonov-Bohm effect which suggests a method of resolving the tension between falsehood and interpretative interest. My first task is to describe and situate the formalism and interpretative problems of electromagnetism. I begin in §2 by sketching the formalism and interpretative problems of gauge theories in general. This allows me to present electromagnetism as a

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\(^1\) Bohr and Rosenfeld’s paper is reprinted as Chapter 1 of Cohen and Stachel 1979. The models which are used to predict the Aharonov-Bohm effect are idealizations in which the classical field acts on the particles, but the particles are not sources of the field. Thus, although they may be useful for describing certain phenomena, they cannot be taken to be accurate representations of our world any more than the models of electromagnetism can be.
gauge theory in §3, and to show how its interpretative problems arise out of its gauge freedom. The presentation of these two sections is quite abstract, but I believe that this approach gives valuable insight into the structure of the classical theory. In §4, I show how attention to quantum mechanics can shift the balance of power among competing interpretations of electromagnetism: there is an ambiguity inherent in the construction of a quantum model of a charged particle moving in an electromagnetic field; distinct interpretations of electromagnetism suggest different ways of resolving this ambiguity; the empirical success of one or another quantum treatment can then have repercussions for our attempts to interpret electromagnetism. In particular, we will see that in the aftermath of the Aharonov-Bohm effect, we are forced to accept that electromagnetism is either indeterministic or nonlocal. Thus we find that the requirement that our false theories mesh in an appropriate way—ontologically as well as empirically—places strong constraints upon our interpretative practice. In the final section of the paper, I attempt to explicate a sense in which false theories tell us about our world, and to show how this fact has important consequences for our understanding of the structure and content of our physical knowledge.

Before beginning my main task, I would like to say a few words about a distinction which will play a fundamental role in what follows. I distinguish between three components of a physical theory: the formalism, the interpretation, and the application. The formalism is some (more or less rigorous) mathematics. This might be of interest to a mathematician with no interest whatsoever in physics. The application is a set of practices which allow one to derive and to test the empirical consequences of the theory. The interpretation consists of a set of stipulations which pick out a putative ontology for the possible worlds correctly described by the theory. Schematically, we can imagine the physical theory being taught in a course for undergraduates: the formalism is developed on the blackboard during lectures; the application is worked out in problem sets and in the lab; the interpretation is fixed via verbal asides which give the students a
heuristic grasp of the content of the theory. A command of all three components will be essential for any student who aspires to full understanding of the theory.

Now, of course, in saying that we misunderstood electromagnetism prior to the discovery of the Aharonov-Bohm effect, I don’t mean to suggest that Einstein misunderstood the formalism of the theory or that Hertz misunderstood its application. These remain fixed as our interpretation changes (or, more properly, they evolve via their own dynamics which need not be directly correlated with interpretative developments). But I do insist that a change in interpretation constitutes a change in understanding.

2. Gauge Theories and Their Interpretation

I am going to sketch a couple of frameworks for doing classical mechanics, and discuss their respective interpretative problems. One is the familiar Hamiltonian formalism. The other is a generalization of the Hamiltonian framework: the language of gauge theories. They share a great deal of their conceptual apparatus. Both Hamiltonian systems and gauge systems consist of triples of mathematical objects: a space, a tensor which gives this space some geometric structure, and a real-valued function on the space, called the Hamiltonian. When equipped with its geometric structure, the space is called the phase space, and its points are thought of as representing the dynamically possible states of some classical physical system (typically a set of particles or fields). The Hamiltonian then determines a class of curves in phase space. These are thought of as representing the dynamically possible histories of the system—if we know which point represents the present state of the system, then a curve through this point passes through points representing the dynamically possible future and past states of the system.

This much, Hamiltonian systems and gauge systems have in common. The difference between them lies in the nature of the geometric structure of phase space. As

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2 The following presentation is meant to be accessible to a general reader. Details are given in the footnotes for those who are familiar with the apparatus of differential geometry.
we will see, the weaker geometric structure of gauge systems brings with it a thorny interpretative problem.

(i) Hamiltonian Systems

The geometric structure of the phase space of a Hamiltonian system is called a symplectic form.\(^3\) It’s chief virtue is the following: specifying a real valued function on the phase space (the Hamiltonian, H) suffices to determine a unique dynamical trajectory through each point of phase space.\(^4\) Figure 1 is an representation of a Hamiltonian system: at the top, we have a phase space; specifying a Hamiltonian serves to determine a unique curve, \(t \mapsto x(t)\), through each point \(x\).

The simplicity of the Hamiltonian formalism makes the following literal approach to interpretation quite attractive. Given a Hamiltonian system, one would like to set up a bijection between points of phase space and dynamically possible states of the system. Then the theory at hand will be deterministic: given a point representing the present state of the system, the dynamical trajectory passing through that point represents the only physically possible past and future of the state.

Typically, it will be quite straightforward to develop such an interpretation. Most of the phase spaces of classical mechanics have the following form. One begins with a space \(Q\), called the configuration space, which represents the possible configurations of some set of particles or fields relative to an inertial frame. One then constructs the so-called cotangent bundle, \(T^*Q\), of \(Q\). This is the set pairs \((q,v)\) where \(q \in Q\) and \(v\) is a vector at \(q\). There is a canonical way of endowing \(T^*Q\) with a symplectic structure, so that it may be viewed as a phase space. Since a point \(q \in Q\) represents a possible

\(^3\) The phase space consists of a manifold, \(M\), equipped with a closed, nondegenerate two-form, \(\omega\).

\(^4\) These curves are the integral curves of the vector field \(X_H\) which solves \(X_H \lrcorner \omega = dH\) (the left hand side of this equation is the contraction of \(X_H\) with \(\omega\)).
(generalized) position of the system, we can think of $v$ as representing the system’s (generalized) momentum.

(ii) Gauge Systems

The geometry of the phase space of a gauge system is determined by a presymplectic form. This notion of geometry is weaker than the symplectic geometry of Hamiltonian systems. For our purposes, the upshot is the following: the phase space of a gauge system has a natural partition into subspaces, called gauge orbits (see the top half of figure 2). The gauge orbits are all of the same dimensionality. Each point, $x$, of phase space lies in exactly one gauge orbit, denoted $[x]$. As in the Hamiltonian case, we specify the dynamics by choosing a real valued function on phase space, the Hamiltonian. However, whereas in the Hamiltonian case there was a single dynamical trajectory through each point of phase space, we find in the gauge theoretic case that there are infinitely many trajectories through each point of phase space. The saving grace is that the family of dynamical trajectories through a given point, although they in general disagree radically about which point represents the future state of the system at a given time, do agree about the gauge orbit in which this point lies. That is: if $t \mapsto x(t)$ and $t \mapsto x'(t)$ are dynamical trajectories which have their origin at the same point $x(0) = x'(0) = x_0$, then we have that $[x(t)] = [x'(t)]$ for all $t \in \mathbb{R}$ (see figure 2). Thus, although the presymplectic geometry is not strong enough to determine a unique dynamical trajectory through each point, it is strong enough to force all of the dynamical trajectories through each point to lie in the same gauge orbit.

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5 A presymplectic form, $\sigma$, on a manifold $N$ is a closed but possibly degenerate two form. It is standard to assume that the dimensionality of the null space of $\sigma$ is the same at all points of $N$. A Hamiltonian system is a gauge system for which the null space is zero dimensional at all points of the phase space.

6 Two points lie in the same gauge orbit iff they can be connected by a curve, all of whose tangent vectors are null vectors of $\sigma$. That is, the gauge orbits are constructed by integrating the null distribution of $\sigma$.

7 That is, we again look at the integral curves of vector fields which solve $X_H \cdot \omega = dH$. We require that $H$ be “gauge invariant” (see below).

8 Indeed let $X$ and $X'$ be vector fields on $N$, and suppose that $X$ solves $X \cdot \omega = dH$. Then $X'$ solves $X' \cdot \omega = dH$ iff $Y = X - X'$ is a null vector field of $\sigma$ (since $0 = X' \cdot \sigma - X \cdot \sigma = Y \cdot \sigma$).
through a given point to agree about which gauge orbit the system occupies at a given
time.

There is an obvious obstacle to the application of gauge theories in classical
mechanics. Hamiltonian systems have well posed initial value problems: if we specify
initial data (i.e. a point, \( x_0 \), in phase space), then there is a unique solution of the
equations of motion for that initial data (i.e. a dynamical trajectory, \( x(t) \)). Now, note that
any given observable classical quantity can be represented by a real-valued function, \( f \), on
phase space (since the state of the system determines the values of all classical
quantities). Thus, if we want to know the value of the quantity at time \( t_1 \), we need only
calculate \( f(x(t_1)) \), where \( x(t) \) is the unique dynamical trajectory through \( x_0 \). But in the
case of a gauge system, there are many dynamical trajectories, \( x(t) \), \( x'(t) \), \( x''(t) \),…, through
each point of phase space, so it is impossible to predict the future value of an arbitrary
function on phase space from the initial state: in general, \( x(t_1) \neq x'(t_1) \), so we expect that
\( f(x(t_1)) \neq f(x'(t_1)) \).

But we know that it must be possible to apply such theories, since many of the
most interesting classical field theories—electromagnetism, general relativity, Yang
Mills—are gauge theories.\(^9\) The solution is quite straightforward: observable quantities
must be gauge invariant. That is, if a real-valued function on phase space is to represent
an observable quantity, then we require that \( f \) be constant on gauge orbits—if \([x]=[y]\),
then \( f(x)=f(y) \). The initial value problem of such a quantity is well-posed: if \( x_0 \) represents
the initial state of the system, and \( x(t) \) and \( x'(t) \) are dynamical trajectories through \( x \), then
\( f(x(t_1))=f(x'(t_1)) \). Thus, despite the ambiguity in the evolution of the states of our gauge
system, there is no ambiguity in the evolution of observable quantities, so long as we
restrict our attention to gauge invariant quantities.

\(^9\) There are also gauge theories which describe the gravitational interaction of point particles. These are of
philosophical interest because they delimit the precise sense in which Mach’s Principle can be implemented
But how can we interpret this formalism? Here are three important interpretative approaches.

The most straightforward option is to mimic the literal approach that worked so well for Hamiltonian systems. This has the advantage of simplicity: one insists that each point of the phase space corresponds to exactly one physically possible state of the system. There is, however, a prima facie grave disadvantage to this approach: if the present state of the system is $x_0$, then in general $x(t_1)$ and $x'(t_1)$ will correspond to distinct possible states of the system; the present will have many possible futures. Thus, literal interpretations render the theory indeterministic.\(^{10}\) Of course, if we supplement this account of the ontology of the theory with an account of measurement which implies that its observable quantities are gauge invariant, then the indeterminism will not interfere with our ability to derive determinate predictions from the theory.

The second interpretative option is to stipulate that each gauge orbit of the phase space represents exactly one physically possible state. In this simply gauge invariant case, our theory will be deterministic. We know that if $x(t)$ and $x'(t)$ are two dynamical trajectories for the same initial data, then $[x(t)]=[x'(t)]$ for all times $t$. Under a simply gauge invariant interpretation, this is equivalent to saying that the present state determines the future and past states. In effect, our theory is construed as a theory of gauge orbits rather than points. There is an elegant construction which makes this precise: one can endow the set of gauge orbits of the gauge system with a symplectic structure and a Hamiltonian.\(^{11}\) The resulting Hamiltonian system is called the reduced phase space. A point in the reduced phase space corresponds to a gauge orbit of the original gauge system; a dynamical trajectory of the reduced phase space tells us which gauge orbits the system passes through, given its initial gauge orbit; a real valued function on the reduced phase space corresponds to a gauge invariant function of the original system. Thus, the

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\(^{10}\) The hole argument of Earman and Norton 1987 is a special case of this observation. See Belot and Earman 1997b,c.

\(^{11}\) Technical complications can make it impossible to carry out this construction.
reduced phase space captures all of the gauge invariant information of the gauge system. A simply gauge invariant interpretation of the gauge system is equivalent to a literal interpretation of the reduced phase space—in both cases, one contends that physical possibilities stand in one-to-one correspondence with the gauge orbits of the original gauge system.

The third option is to adopt a *coarse grained gauge invariant* interpretation, according to which the representation relation between gauge orbits and physically possible states is many-to-one. Of course, since there is no physical difference between points in the same gauge orbit, coarse grained gauge invariant interpretations are deterministic (by the argument of the preceding paragraph).

This terminology provides us with a framework for discussing the interpretative possibilities for gauge theories. We can now see that it is an immediate consequence of the formalism of gauge theories that every such theory admits multiple interpretations. Settling on an interpretation is an important part of understanding the theory, since rival interpretations will disagree about whether or not the theory is deterministic, and about amount of the structure of the phase space which is physically relevant.

This is likely to seem somewhat implausible at this stage. After all, isn’t it fairly clear that the preferred interpretation of a given gauge theory is a simply gauge invariant interpretation? And, since this is the same as a literal interpretation of the reduced phase space, and it is supposed to be straightforward to construct a literal interpretation of a Hamiltonian system, doesn’t it follow that it should be straightforward to formulate such an interpretation? If this is so, then literal and coarse grained gauge invariant interpretations of gauge theories will never arise in practice.

I grant that simply gauge invariant interpretations are, ceteris paribus, to be preferred. But I maintain that things are not always equal: sometimes it turns out that the available simply gauge invariant interpretations are less plausible than their competitors. We will see an example of this below, in the case of electromagnetism. The problem is
that, although the task of finding a simply gauge invariant interpretation for a gauge theory does indeed reduce to the task of finding a literal interpretation of the theory’s reduced phase space, it does not follow that it is straightforward to find a plausible interpretation of this Hamiltonian system. Literal interpretations of Hamiltonian systems are impeccable when the phase space has the structure of the set of positions and momenta of some set of fields or particles. But, in general, there is no guarantee that the reduced phase space of a gauge system will have such a structure.

3. Interpreting Electromagnetism

The language of gauge theories provides the setting for a very elegant formulation of electromagnetism. Let physical space be modeled by some three-dimensional Riemannian geometry, \( S \).\(^{12}\) Then the phase space of electromagnetism consists of pairs, \((A(\xi), E(\xi))\), of vector fields of on \( S \), subject to the condition that \( \text{div } E = 0 \). That is, each point in the phase space of electromagnetism gives us a pair of maps, \( A \) and \( E \), each of which assigns a three-vector to each point of \( S \). We call \( A \) the vector potential, and \( E \) the electric field. This infinite dimensional phase space comes equipped with a natural presymplectic structure.

The gauge orbits of the phase space have the following structure: \((A,E)\) and \((A',E')\) belong to the same gauge orbit iff for all \( \xi \in S \), we have that \( E(\xi) = E'(\xi) \) and that there exists a function \( \Lambda: S \to \mathbb{R} \) such that \( A(\xi) = A'(\xi) + \text{grad } \Lambda(\xi) \). The Hamiltonian for electromagnetism is just \( H = \int_S (|E|^2 + |\text{curl } A|^2) d\xi \). The dynamical trajectories are then determined by the following equations:

\[
\begin{align*}
\dot{A} &= -E \\
\dot{E} &= \text{curl } (\text{curl } A) \quad (*).
\end{align*}
\]

\(^{12}\) The metric structure of \( S \) plays a hidden role in what follows: it allows us to define div, grad, and curl for non-Euclidean spaces.
These are Maxwell’s equations. Of course, these equations do not uniquely determine the evolution of the variables of electromagnetism: they do so only upto gauge. Thus, if we fix an initial point in phase space, \((A_0, E_0)\), then we find that the value of \(E\) is determined for all times, past and future. But the value of \(A\) is only fixed upto the addition of the gradient of a scalar function on space: if \((A(t), E(t))\) and \((A'(t), E'(t))\) are two solutions for our given initial data, then for all \(t\), \(E(t) = E'(t)\) and there exists a scalar \(\Lambda(t)\) so that \(A(t) = A'(t) + \text{grad } \Lambda(t)\). Maxwell’s equations do not determine the future value of \(A(t)\), but they do determine the gauge orbit in which \(A(t)\) lies.

I will present three interpretations of electromagnetism, corresponding to the three strategies for interpreting gauge theories which were canvassed in the preceding section.\(^{13}\) It is helpful to have in mind some desiderata that we would like any interpretation of electromagnetism to fulfill.

First of all, of course, we would prefer that our interpretation render the theory deterministic: that is, we would like to find a gauge invariant interpretation of electromagnetism.

Second, we would like our theory to be local. Here I draw a distinction between two types of locality.\(^ {14}\)

(I) Synchronic Locality: The state of the system at a given time can be specified by specifying the states of the subsystems located in each region of space (which may be taken to be arbitrarily small).

(II) Diachronic Locality: In order to predict what will happen \(\text{here}\) in a finite amount of time, \(\Delta t\), we need only look at the present state of the world in finite neighborhood of \(\text{here}\), and the size of this neighborhood shrinks to zero as \(\Delta t \to 0\).

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\(^{13}\) There are a few other interpretations of electromagnetism—see Brown 1994, Cao 1988, and Kennedy 1993. Considerations of space lead me to omit discussion of these here. None of them serves to undercut the themes developed below.

\(^{14}\) These are closely related to, but not identical with, the notions of seperability and locality which are employed in the literature on the Bell inequality.
The first principle is meant to express the intuition that the properties of classical physical systems should be reducible to the properties of their constituent parts. Classical fields, thought of as assignments of properties to the points of physical space, are paradigm examples of synchronically local systems—at each here and now the field has a state, and the state of the field itself is nothing but the sum of its states at each of these here and nows. Thus it makes sense, for instance, to speak of the state of the field in a given region, without reference to anything far removed from the region under consideration.

An object is local in the (strictly stronger) diachronic sense if its evolution in time is such that in order to know what its future state will be here, we need only know its present state in some finite hereabouts. Not every classical field is diachronically local. The Newtonian gravitational field is a well known example of a diachronically nonlocal object—since gravitational effects propagate with infinite velocity, it is necessary to know the gravitational state everywhere in order to know exactly what will happen next here.

On the other hand, one expects electromagnetism to be both synchronically and diachronically local. It is, after all the theory of the electromagnetic field and so should be synchronically local. Furthermore, Maxwell’s equations determine that electromagnetic radiation propagates at a fixed speed. This seems to imply immediately that electromagnetism is a diachronically local theory—since it will take some known finite amount of time for influences over there to reach here, I need only take into account what is happening over there when reckoning what will happen here if I am interested in sufficiently large \( \Delta t \).

We will see, however, that electromagnetism is diachronically local under only one of the three interpretations discussed below. Even worse, the theory is not even synchronically local according to one of these interpretations!

Interpretations
(1) *The vector potential as a physical field.* Under this first interpretation, one maintains that the vector potential, \( A \), represents a physically real field on physical space, \( S \). Most dramatically, one could maintain that the vector potential represented the velocity of a material ether. Then of course, the electric field, \( E = -\dot{A} \), would correspond to the acceleration of the material ether.\(^{15}\) This gives us a literal, hence indeterministic, interpretation of the gauge theoretic formulation of electromagnetism: each pair \((A, E)\) satisfying \( \text{div} \ E = 0 \) represents a distinct dynamical state of the ether, and two solutions, \( A(t) \) and \( A'(t) = A(t) + \text{grad} \Lambda(t) \) for the same initial data represent two physically distinct physical histories of the ether (according to \( A \), this bit of ether ends up here; according to \( A' \) it ends up over there).\(^{16}\)

This interpretation is, of course, synchronically local, since the state of the field reduces to the state of the field at each point of space. But it is diachronically nonlocal: changing the magnetic field in a given region can change the vector potential throughout space instantaneously—changes in the vector potential can propagate with infinite velocity even though electric and magnetic radiation propagates with a finite velocity. So in order to know what will happen to us next, we need to know what is presently happening arbitrarily far away. We will see an example of this phenomenon in the next section.

(2) *The traditional interpretation.* One would like to avoid the conclusion that electromagnetism is indeterministic. Thus, one would like to look for a gauge invariant interpretation of electromagnetism. Here the most obvious option is the familiar one. We begin by defining the magnetic field to be \( B \equiv \text{curl} \ A \). The value of the magnetic field at a point of physical space is a gauge invariant quantity, since \( \text{curl} \ A = \text{curl} \ (A + \text{grad} \Lambda) \).

\(^{15}\) This is only one step removed from historical reality: in Maxwellian electrodynamics the current was sometimes interpreted as representing the acceleration of the ether. See Buchwald 1985, p. 24.

\(^{16}\) As usual, we can nonetheless maintain that the theory is predictable if we can argue that only gauge invariant quantities are measurable. If, for instance, our ether were an imponderable fluid, which interacted with ordinary matter only via electric and magnetic phenomena, then we would be unable to empirically distinguish between points of phase space which lie in the same gauge orbit.
Together, E and B capture almost all of the content of our gauge theory of electromagnetism.\textsuperscript{17} Indeed, it is not difficult to show that the equations (\textsuperscript{*}), together with the constraint \( \text{div} \ E = 0 \) and the identity \( B = \text{curl} \ A \), are equivalent to the familiar vacuum Maxwell equations:

\[
\begin{align*}
\dot{B} &= -\text{curl} \ E \quad \text{div} \ B = 0 \\
\dot{E} &= \text{curl} \ B \quad \text{div} \ E = 0
\end{align*}
\]

\textsuperscript{\textsuperscript{*\textsuperscript{*}}}. The initial value problem for (\textsuperscript{\textsuperscript{*\textsuperscript{*}}}) is well-posed: if we specify E and B at an initial time, then there is a unique solution of (\textsuperscript{\textsuperscript{*\textsuperscript{*}}}) which gives E and B for all future times.

Thus, if we stipulate that the ontology of electromagnetism consists of physically real electric and magnetic fields, then we have an interpretation which is deterministic (being gauge invariant) and is clearly synchronically local (being based on fields).\textsuperscript{18} Furthermore, this interpretation is diachronically local: Maxwell’s equations imply that the electric and magnetic fields propagate at the speed of light—in order to know what will happen \textit{here} in \( \Delta t \) seconds, we need only consider the electromagnetic state of a sphere of radius \( \Delta t \times c \).

There remains one further question: Is this interpretation simply gauge invariant or coarse grained gauge invariant? It turns out that the answer depends upon the topology of physical space, S. If the topology of space is trivial, in the sense that S is simply connected, then the reduced phase space of electromagnetism is just the set of divergence free electric and magnetic fields on S.\textsuperscript{19} In this case, our interpretation may be viewed either as a literal interpretation of the reduced phase space or as a simply gauge invariant interpretation of the original gauge system. If, however, S is topologically nontrivial, then the reduced phase space has a more complex structure (see below). Then the traditional

\textsuperscript{17} The significance of this ‘almost’ will be made clear below.
\textsuperscript{18} Under this regime the vector potential becomes a useful mathematical fiction, with no physical content beyond that encoded in the magnetic field.
\textsuperscript{19} A space is said to be \textit{simply connected} if every closed curve in the space may be contracted to a point without leaving the space. Otherwise, it is said to be \textit{multiply connected}. Thus, in two dimensions the sphere and the plane are simply connected, while the cylinder and the torus are multiply connected (imagine drawing a circle which goes around the circumference of the cylinder or torus—such a curve cannot be contracted to a point without leaving the surface).
interpretation counts as coarse grained gauge invariant: a number of gauge orbits correspond to each configuration of the electric and magnetic fields. In this case, there will be points in phase space, \((A,E)\) and \((A',E')\), such that \([A,E]\neq[(A',E')]\) but \(A\) and \(A'\) correspond to the same magnetic field. We will see in the following section that this is the downfall of this otherwise very attractive interpretation.

(3) **Holonomies.** One would like an interpretation of electromagnetism which was simply gauge invariant, no matter what the topology of physical space. This is possible, but requires some ingenuity—and some sacrifices. The first step is to observe that although the value of the vector potential at a given point of physical space is not gauge invariant (since \(A(\xi)\neq A(\xi) + \text{grad } A(\xi)\)), the integral of \(A\) around a closed curve, \(\gamma\),

\[
\text{h(}\gamma\text{)}= \exp \left( \oint_{\gamma} i A_a(\xi) d\xi^a \right)
\]

is gauge invariant. In fact, if we substitute a vector potential, \(A'\), into the integral which defines \(\text{h(}\gamma\text{)}\), then this quantity is unchanged if and only if \(A'\) is in the same gauge orbit as \(A\). \(\text{h(}\gamma\text{)}\) is called the *holonomy* around \(\gamma\), and is a complex number of unit modulus.

It is a remarkable fact that we can construct the reduced phase space of electromagnetism by taking the set of holonomies around all curves in physical space as the basis for our configuration space. That is, if we call the set of closed curves in physical space *loop space*, each point in our configuration space is just a map from loop space to the complex numbers of unit modulus. Let us call these maps holonomy maps. The value of a given holonomy map on a given loop is just the holonomy around that loop. We proceed to construct the phase space by building the cotangent bundle, which is just the set of pairs consisting of a holonomy map and a divergence free electric field. After we impose the canonical symplectic structure, and the correct Hamiltonian, we end up with a Hamiltonian system which is the reduced phase space formulation of electromagnetism.

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20 See Wu and Yang 1975 for an influential presentation of this approach.
We would like to give a literal interpretation of this reduced phase space formalism. We know that this can be done in terms of the electric and magnetic fields iff physical space is topologically trivial. But in general, the topology of S is nontrivial, and this move is not available—we cannot identify the space of holonomy maps with a space of tensors on S.

It is, nonetheless, possible to formulate an interpretation of electromagnetism which is simply gauge invariant no matter what the topology of space. The reduced phase space formulation of electromagnetism suggests that a state of the electromagnetic field should be thought of an assignment of real numbers to closed curves in space (holonomy), together with an assignment of vectors to points of space (electric field). This requires a revision of our notion of field. We can no longer think of the electromagnetic field as simply being an assignment of properties to points of space. Rather, we must also consider closed curves in space to be carriers of the electromagnetic predicates. This interpretation is, of course, deterministic, since it is simply gauge invariant. But it is also synchronically (and hence also diachronically) nonlocal, since specifying the electromagnetic state of any given region of physical space entails knowing the holonomy around every loop in space, and hence mentioning regions of space arbitrarily far away from the one under consideration.

These, then, are our three interpretations. Here we find ourselves in a situation of the sort alluded to at the end of the previous section. Ceteris paribus, we would prefer a simply gauge invariant interpretation of our theory. But in the case at hand this is awkward: unless physical space has a very special topological structure, the reduced phase space of electromagnetism cannot be viewed as the space of positions and momenta of some set of particles or fields relative to physical space. Thus, simple gauge invariance can only be had at the price of a revision our intuitions about classical fields.
In this context, it seems clear that interpretation (2) is to be preferred. It is the only interpretation which is both deterministic and diachronically local. When S has a non-trivial topology, it is true, this traditional interpretation has a vice: since it is coarse grained, it loses information which is stored in the other interpretations. As a result, this interpretation is highly vulnerable to empirical refutation: it is an empirical question whether or not there exist measurable physical quantities which distinguish between each pair of gauge orbits. If there were any such quantities, no coarse grained gauge invariant interpretation would be tenable. Within the realm of classical physics, however, (2) is vindicated—there are no phenomena which allow one to distinguish between two gauge orbits [A] and [A'] which correspond to the same magnetic field. Thus, there are no grounds internal to electromagnetism upon which to criticize the traditional interpretation.

4. Quantization and the Aharonov-Bohm Effect

The story of the Aharonov-Bohm effect is normally told in a very abbreviated form. Something like: “The quantum treatment of a charged particle in an external magnetic field shows that it is possible to distinguish between vector potentials which correspond to the same magnetic field. So quantum mechanics shows that the vector potentials of electromagnetism are physically real.” This sort of account gives the correct flavor. But it is also quite misleading: if the vector potential is physically real in the way that the electric field is supposed to be, then electromagnetism is indeterministic (as in interpretation (1) of the previous section)—but few commentators mean to commit themselves to such a view! In this section, I will present a careful rational reconstruction of the way in which the Aharonov-Bohm effect bears upon the considerations of the previous section.21

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21 See Healey 1997 for a complementary discussion of the bearing of the Aharonov-Bohm effect upon approaches to interpreting quantum mechanics.
The story has a complex structure, so I have broken it down into several parts. The first part consists of a generic account of how interpretative beliefs about classical theories bear upon quantization, and vice versa. In the second, this apparatus is applied to the special case of a charged particle in a static magnetic field. Finally, in the third part, the Aharonov-Bohm effect makes its appearance.

(i) Quantization and Interpretation

Quantization is a technique for producing quantum versions of classical theories. Here I will focus on canonical quantization, where the initial input is a classical theory in Hamiltonian form, and the output is a quantum theory which—one hopes—has the original classical theory as its classical limit. It is important to emphasize that neither quantization nor the taking of a classical limit is entirely straightforward. The details of either sort of process vary greatly from case to case. Thus there is considerable play at either end whenever we attempt to set up a correspondence between classical and quantum systems. This freedom in bridging the classical-quantum divide is closely related to an interplay which exists between interpretative issues at the classical and quantum levels. This works as follows.

It is well known that all infinite dimensional Hamiltonian systems have infinitely many quantizations. That is: there are infinitely many quantum field theories which are quantizations of any given classical field theory. This fact has received some attention from philosophers of physics. But it has tended to be regarded as one pathology among many in the foundations of quantum field theory. What has received little (no?) attention from philosophers, is the fact that the same situation is endemic among finite dimensional systems. Indeed, any Hamiltonian system with a topologically nontrivial phase space has infinitely many quantizations. Thus, the ambiguity involved in quantization cannot be

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22 The physically inequivalent quantizations of a given Hamiltonian system are parameterized by the cohomology group $H^1(M,U(1))$, where $M$ is the classical phase space, and $U(1)$ is the group of complex numbers of unit modulus. This group is a topological invariant—it is determined by the topology of the
dismissed as a pathological feature of a theory which is still under development—it is to be found everywhere in ordinary quantum mechanics.

We have to ask ourselves what attitude to take towards inequivalent quantizations of a given classical theory—how many of them are physically significant? Often there will be a distinguished quantization.\(^{23}\) In this case it will be tempting to maintain that it alone should count as an admissible quantum treatment of the phenomenon under investigation. On the other hand, it can happen that one has reason to believe that there are many acceptable quantizations of the system (each appropriate for modeling a distinct physical situation), or that the structure of the classical system does not single out a preferred quantization (as in quantum field theory on curved spacetime).

Ultimately, of course, one hopes that empirical considerations will determine which quantizations should be taken seriously. But one sometimes finds oneself in a situation where available data underdetermine the question—indeed this situation is probably the norm at the frontiers of theoretical physics. Here there are a number of epistemic resources which could be mobilized to fill the gap. Among these are interpretative beliefs about other theories.

Thus, one’s interpretative beliefs can shape one’s judgments as to the relevance of certain quantizations or approaches to quantization. Conversely, one must accept that one’s interpretative beliefs are open to revision in light of the empirical success of the approaches to quantization which they suggest. If one’s interpretation of a given classical theory suggests that quantization A is superior to quantization B, then one is bound to revise one’s interpretative judgments if it turns out that A is empirically untenable.

This is, I maintain, the best way of thinking of the import of the Aharonov-Bohm effect: we come to quantum mechanics with prior interpretative beliefs about classical phase space. For our purposes, it suffices to assume that this cohomology group is nontrivial iff the classical phase space is multiply connected, although this is not quite true.

\(^{23}\) Indeed, this is always the case for quantizations of finite dimensional systems, since the group $H^1(M, U(1))$ always has a preferred element—namely, the identity.
electromagnetism; these suggest a particular approach to quantizing classical systems involving electromagnetic fields; when this doesn’t pan out, we are obliged to revise our understanding of electromagnetism.

(ii) Quantizing the Charged Particle in a Magnetic Field

The Aharonov-Bohm effect involves a quantum charged particle moving in a static magnetic field. So we are interested in quantizing the classical treatment of such a particle.\(^{24}\)

In order to construct the classical treatment of the particle, we proceed as follows. We let \(S\) be the region of physical space in which the particle is free to move. The configuration space of the particle is isomorphic to \(S\). The phase space of the particle is just the set of possible positions and momenta for such a particle (the cotangent bundle of \(S\)), endowed with a symplectic structure which differs from the symplectic structure for a free particle by a factor which is determined by the magnetic field. Finally, the Hamiltonian, as in the treatment of a free particle, is just the kinetic energy.

This system will have a unique quantization iff \(S\) is topologically trivial. We will be interested in a case where \(S\) is ordinary Euclidean three dimensional space with the \(y\)-axis removed, and the electric and magnetic fields are zero. Thus, our model of the charged particle moving in \(S\) will be identical to our model of a free particle moving in \(S\). This model has infinitely many quantizations. These quantizations can be put in a one to one correspondence with the set of complex numbers of unit modulus.\(^{25}\) That is, specifying a complex number, \(z\), with \(|z|=1\) determines a quantization \(Q(z)\) of our classical system. Of course, one possibility stands out: the quantum system \(Q(1)\). Intuitively, the quantization \(Q(z)\) predicts that a quantum charged particle which is transported around the \(y\)-axis will have its phase shifted by a factor of \(z\).\(^{26}\)

\(^{24}\) I.e. the particle is treated quantum mechanically while the field is treated classically.

\(^{25}\) This follows from the topology of the phase space, \(T^*S: H^1(T^*S,U(1))=U(1)\).

\(^{26}\) We would have to be a bit more careful at this point if we were working with \(B\neq0\), and specify a
Now, in the absence of empirical data, what attitude should one take towards these various quantizations? If we believe in a simply gauge invariant or literal interpretation of electromagnetism, we will be open to the idea that each of the quantizations of the charged particle represents a genuine physical possibility. After all, the Hamiltonian representation of this particle contains only partial information about the electromagnetic state: the Hamiltonian $H$ contains no information at all, and the symplectic form contains information about the magnetic field alone. But in the case at hand, where $S$ is multiply connected, we know that specifying $B$ is inadequate—the true electromagnetic state of the world contains considerably more information, since many gauge orbits of the phase space correspond to the same $B$. In the present case this excess information can be encoded as a single complex number of unit modulus—the holonomy around a loop which circumnavigates the y-axis. In light of this fact, it would be quite natural for someone who espouses interpretations (1) or (3) of electromagnetism to adopt the following line. In order to specify the electromagnetic state of the world, one must specify the holonomy around a loop around the y-axis as well as the values of $E$ and $B$; since our Hamiltonian treatment of the charged particle in an static magnetic field doesn’t contain this additional information, it is no surprise that there are multiple quantizations and that these are in one-to-one correspondence with the possible values of the holonomy: this just means that there is one quantization for each possible electromagnetic state with $E=B=0$. Each of these quantizations should be taken seriously, since each is appropriate for a distinct physical situation.

If, on the other hand, we accept the traditional interpretation of the theory—according to which the electric and magnetic field are the only physical realities,
and any information contained in the reduced phase space of electromagnetism which is not determined by the values of these fields is descriptive fluff—then we will not believe that there are any hidden variables which could determine which of the quantizations of our classical system is the physically correct one. Our attitude will be: having built all of the physically relevant information into our classical model of the particle moving in a magnetic field, we have no reason to believe that it admits of more than one physically realistic quantization. This line of argument will seem especially convincing when it is recalled that we use the same classical model to represent a free particle in region S—and of course we believe that in this latter case there is a unique physically correct quantization.

Thus the coarse grained gauge invariant interpretation (2) leads to a different approach to quantizing the charged particle in a magnetic field than that suggested by the other interpretations of electromagnetism. And here we have an empirical question: What would a charged quantum particle do in a situation like this? If we find that distinct gauge orbits of vector potentials which correspond to different holonomies but to the same magnetic field lead to different interference patterns, then the coarse grained gauge invariant interpretation is sunk—it leads to an empirically inadequate quantum theory. Otherwise, it survives this test, and will remain the interpretative approach of choice.

(iii) The Aharonov-Bohm Effect

It isn’t possible to subject interpretation (2) to a direct empirical test—we simply do not know enough about the topology of physical space. And even if we did, it is unlikely that we would be able to detect these sorts of effects on the cosmological scale. There is, however, a tabletop experiment which is widely regarded as refuting interpretation (2). This experiment was first suggested by Aharonov and Bohm in 1959, and was carried out shortly thereafter. The idea is to restrict an electron to a region which
can be treated for all practical purposes as being topologically nontrivial. One should then be able to see which quantizations are physically relevant.

The experiment works as follows. One constructs a solenoid—a conducting wire coiled around a cylinder—whose length is long compared to the wavelength of the particle under consideration. When current runs through the solenoid, a magnetic field is created inside the device, but the magnetic field external to the solenoid is unaffected by turning the thing on.28 Now suppose that we want to model the behavior of a classical charged particle in the field-free region external to, but near, a solenoid. What manifold should we use as our configuration space? We could use S′, some region of physical space which includes the solenoid. But then our symplectic form will have to encode information about the state of the magnetic field inside the solenoid. This will be formidably complicated when the solenoid is operational.

There is, however, another option for describing the system which promises to be much simpler. We can assume that the walls of the solenoid are impenetrable. Now, under any reasonable interpretation of electromagnetism, the magnetic field is synchronically local (more on this below). So it seems that we should not have to know anything about the interior of the solenoid in order to model the behavior of the particle. Thus, we should be able to take S as our configuration space, where S is just S′ with the points occupied by the solenoid deleted. For all practical purposes, we can treat the solenoid as being infinitely long. The resulting phase space is of the form discussed in subsection (ii) above. Since the classical particle is indifferent to the state of the field inside the solenoid, we always use the same Hamiltonian system, no matter how much current is running through the solenoid.

28 It follows that the vector potential propagates with infinite high velocity: the holonomy around a closed curve which loops around the solenoid once is equal to the magnetic flux through the solenoid; if we switch the thing one, then the values of the vector potential at some point arbitrarily far away must instantaneously change.
When we want to construct our quantum theory of the charged particle moving in the region external to the solenoid, we find ourselves in the situation described in the previous subsection: the inequivalent quantizations of our system are parameterized by the complex numbers of unit modulus, and we want to know how many of them model physical possibilities. But now we have a manageable experimental question: What happens when we shoot a beam of electrons by a solenoid? Experiment reveals that the interference pattern which results does depend on whether or not the solenoid is operational. In fact, it turns out the quantization which yields the correct predictions for a given experimental situation is \( Q(z) \), where \( z \) is the holonomy of a loop which circumnavigates the solenoid.

Thus interpretation (2) of electromagnetism is in trouble. It led us to expect that only one of the quantizations would be empirically interesting. This turns out to be false. This, in turn, casts considerable doubt on the position that the electric and magnetic fields encode all of the physically relevant information. Interpretations (1) and (3), on the other hand, allow that the holonomies around closed curves carry genuine physical information over and above that contained in a specification of the fields. They are thus able to easily account for the Aharonov-Bohm effect.

Where does this leave interpretation (2)? Above, I noted that it is permissible to employ \( S \) as the configuration space of the classical system only if we believe that the behavior of the classical charged particle depends only on the field values where it is, and not on the field values in the interior of the solenoid. Now, it is possible to alter interpretation (2) so that the magnetic field acts where it is not. In doing so, one gives up synchronic—and hence also diachronic—locality. But one is able to thus account for the Aharonov-Bohm effect. Presumably, few are going to be happy with this nonlocal version of interpretation (2).

For the vast majority of physicists, this has provided sufficient reason to turn away from interpretation (2), and to take seriously Aharonov and Bohm’s conclusion that
we should regard the vector potential “as a physical variable” (1959, p. 491). But what exactly does this mean? Is it an endorsement of interpretation (1) or of interpretation (3)? Aharonov and Bohm themselves seem to have had interpretation (1) in mind.29 This was, however, before it was shown that one could give a simply gauge invariant interpretation in terms of holonomies. Today I think that almost anyone who takes the care to distinguish between interpretations (1) and (3) would endorse the latter: people seem to be more willing to make the move to thinking of fields as properties of loops—thus sacrificing even synchronic locality—than they are to sacrifice determinism.30

5. Conclusion

The articulation of the content of our physical knowledge is one of the chief tasks of philosophy of physics. Much of this work is interpretative in nature: one wants to know what sort of world this or that physical theory could possibly describe. This interpretative work is normally carried out on a piece by piece basis: philosophers of physics tend to specialize in a single theory, X, and to face the interpretative problems of X under the assumption that X is true. This often leads to a fruitful and interesting discussion of the interpretative problems of X, which tells us quite a bit about ways the world could be, were X true.

If X were true, this would of course be an impeccable methodology. But, as a matter of fact, X is not true—no extant theory is both fully relativistic and fully quantum. None of our physical theories are candidates for truth: we know enough about their shortcomings to know that our world cannot be one of the worlds that they portray. I maintain that the interpretative enterprise is nonetheless worthwhile. We want to

29 They gloss their comment quoted above by saying that “[t]his means that we must be able to define physical difference between two quantum states which differ only by a gauge transformation.” I read this as indicating that they believe that points in the same gauge orbit correspond to distinct physical situations.

30 One factor here is surely that holonomies and loops have provided a fruitful framework for the quantization of gauge theories such as electromagnetism, general relativity, and Yang-Mills theories. This sits well with the themes developed in the next section.
understand the content of our physical knowledge, and finding interpretations of extant physical theories is part of that enterprise. The falsehood of our theories does not cast doubt on the worth of our interpretative investigations. But it does give us reason to question the standard piecemeal methodology.

The discussion of the preceding sections shows that distinct interpretations of electromagnetism constitute different ways of understanding the theory: according to some interpretations the theory is deterministic, according to others it is nonlocal. The significance of these divergent approaches becomes clear when we examine the conceptual relationships between electromagnetism and quantum mechanics: different interpretations of the classical theory of the electromagnetic field suggest distinct approaches to resolving the ambiguity inherent in the quantization of classical models of the behavior of charged particles. Fairness then requires us to recognize that the empirical success of one or another of these approaches of quantization bears upon our understanding of electromagnetism. Similarly, the question of whether the general covariance of general relativity should be understood as a principle of gauge invariance forms a bridge between the interpretative problems of general relativity (what is the nature of the existence of spacetime points?) and those of quantum gravity (do time and change exist at the fundamental level?).

Different approaches to understanding the general covariance of general relativity are associated with different solutions to the interpretative problems of classical and quantum gravity. Thus, the empirical vindication of a given approach to quantizing gravity may have repercussions for our understanding of classical spacetime ontology.

Examples such as these force us to conclude that theories cannot be interpreted in isolation from one another: understanding intertheoretic relations is a crucial component of the articulation of the content of individual physical theories. Thus, there is another way of gleaning information from X about what the world is like, beyond pretending that

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31 See Belot and Earman 1997b,c.
X is true and that the world is as it would have to be for this to be the case. One can also look at the relation between X and some other theory Y which describes phenomena of our world which are related to those described by X; and by looking at how the descriptions of X and Y mesh, one can discover quite a bit about what X is telling us about our world.

Consideration of theories in relation to one another rather than in isolation, should lead to a richer appreciation of the content of our physical theories. In order to precede, we must develop a way thinking about the structure and content of physics which provides a useful framework for discussion intertheoretic issues, as well as intratheoretic ones. This should start from a recognition that at the present time we have a number of theories on the books: classical mechanics, statistical mechanics, electromagnetism, quantum mechanics, quantum statistical mechanics, quantum field theory, special relativity, general relativity, (parts of) quantum gravity, and so on. These theories tell us about very different worlds. Some are populated by particles, some by fields. In some the spatiotemporal structure is an unchanging backdrop, in some it is an active and changing participant. It is an important fact that, despite the nontrivial overlap between the domains of applicability of pairs of these apparently incompatible theories, we find that peaceful coexistence rather than competition is the rule. Each of these theories informs us about our world, despite their profound divergence of opinion concerning ontology.

So we have a network of independent theories. This network is often described as a hierarchy, the idea being that some theories are more fundamental than others. In fact, a web or a lattice would be a more appropriate metaphor here, since theories often have more than one limit—special relativity is the curvature→0 limit of GR, while the Newton-Cartan theory is its c→∞ limit. It is possible to speak of one theory being more fundamental than another, only so long as we don’t make the mistake of assuming that ‘more fundamental’ gives us a linear ordering of the class of theories.
Each of our theories is empirically adequate within its own domain of applicability, but shares parts of this domain with other theories which also save their own phenomena. Given this situation, it seems essential to demand for every pair of overlapping theories an assurance that they really do agree on their shared domains. The correspondence principle, for example, can be understood along these lines: as requiring that quantum mechanics be able to account for the empirical adequacy of classical mechanics.\(^{32}\)

The following picture emerges. Each of our physical theories is part of a network theories which stand in subtle relations to one another. Each theory contributes to our understanding of the world not only in virtue of its internal structure and empirical adequacy, but also because of the relations in which it stands to other theories. Indeed, we have seen that the content of a given theory may depend upon how it is situated in the network—quantum considerations help to fix the meaning of the terms of electromagnetism, the substantival-relational debate about the spacetime of general relativity is intimately connected with ongoing work on quantum gravity. In terms the web metaphor: if one looks only at the nodes of the web, one gains only partial information; in order to grasp the full content of the fact that this web is useful for describing our world, one must also look at the strands which join these nodes. More graphically—if somewhat distastefully—if one wants to survey the shape of an object caught in a net, it is useful to note how the threads bulge, as well as where they meet.

In the case of electromagnetism, the concrete payoff of this approach is that one can clarify the interpretative status of electromagnetism by looking at the relation

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\(^{32}\) It is important to emphasize that one cannot expect too much of the correspondence principle and its generalizations. It is not in general possible to show that as one takes a classical or non-relativistic limit the ontology of a given theory goes over into the ontology of the limiting theory (see Batterman 1995, Berry 1994, and Rohrlich 1988). And this should not bother us if we keep in mind that both the given theory and the limit theory are false. Similarly, it is important to keep in mind that the correspondence principle requires only that the claim that each of the theories is empirically adequate be consistent, and not that the theories be empirically equivalent. This makes it possible, for instance, to maintain the truth of the correspondence principle, even in the face of the existence of periodic quantizations of chaotic classical systems (Belot and Earman 1997a).
between the Hamiltonian system which models a classical charged particle in a magnetic field and its quantization. Here, constructing a new node in the web of theories (quantum mechanics) and linking this node to its neighbor (classical mechanics, via quantization and classical limits) has necessitated adjusting the position of the web elsewhere (we have been forced to relinquish an attractive interpretation of electromagnetism, in light of the conceptual structure of the relation between classical and quantum mechanics, and the results of empirical investigations).

All of this can be re-expressed in the idiom of possible worlds. The set of possible worlds includes all of the worlds which are correctly described by any possible physical theories, and then some. We do not know which world is ours, but we have some ideas about which worlds it cannot be. If we know that our world is in some set of \( S \) of possible worlds, and subsequently determine that we are justified in believing that it is in fact in some proper subset \( S' \) of \( S \), then we have discovered something about our world. I will now argue that to discover something about either the empirical adequacy of our network of physical theories or the interpretative status of these theories, is to learn something about our world in this sense.

Let us suppose that the structure of the set of possible worlds is such that if \( A \) and \( B \) are worlds correctly described by classical mechanics, but are otherwise very different from one another, and \( C \) is a world correctly described by quantum mechanics, which is empirically indistinguishable from \( A \) over a wide range of possible observations, then \( C \) will be closer to \( A \) than \( B \) is.\(^{33}\) Indeed, I will suppose that for certain classical worlds (such as a world containing only a single free particle), we can find quantum worlds which are arbitrarily close to this world (worlds containing wave packets with some appropriate structure).

\(^{33}\) That is: I impose a very different topology on the set of possible worlds from that employed by Lewis 1986. For him, closeness is determined by qualitative similarity, construed as agreement of perfectly natural properties. But \( C \) will share very few perfectly natural properties with \( A \), since the sciences which describe them use very different vocabularies.
For each of our false physical theories, we have some idea of their domain of empirical adequacy. Thus each of our physical theories picks out an open set in the space of possible worlds: the worlds for which the theory at hand is empirically adequate for the appropriate range of phenomena. Let’s call this set the theory’s *empirical set*. This set will include many worlds which are described accurately by the theory. But it will also include many worlds of which the theory is false, but for which the theory happens to make the correct predictions when restricted to the appropriate type of phenomena. Thus, the empirical set of classical mechanics will include many worlds which are in fact correctly described by quantum mechanics or general relativity. It will, of course, also include many worlds which are not correctly described by any extant theory. Our own world is one such, and is contained in the empirical set of each of our physical theories.

One way that our false theories tell us about our world is that their empirical sets cut down the possibilities for which world could be ours: we know that our world lies in the intersection of the empirical sets of each of our theories. Adding a new (false) physical theory, or finding out that the empirical set of an extant theory is smaller than previously believed, shrinks the size of this intersection, and hence narrows the possibilities.

Another way that false theories tell us about our world emerges from our discussion of the Aharonov-Bohm effect. There we saw that it was important that our interpretations of electromagnetism and quantum mechanics should mesh in an appropriate way. This principle can be generalized as follows: if theories A and B are such that interpretation I of theory A supports or motivates interpretation I' of theory B, then we require that if the closest worlds to our own of which theory A are true are ones described by interpretation I, then the closest worlds of which theory B is true should be described by interpretation I'. Thus, discovering something about A can change our favored interpretation of B, and vice versa.
This is a strong principle of interpretative methodology. It helps us to radically decrease the size of the set of possible worlds which could be our own: it tells us that there are certain worlds which lie in the intersection of the empirical sets of all of our theories which cannot be our world. Thus, interpretation imposes a constraint on the content of our total science which goes beyond that imposed by empirical adequacy. It is in this sense that interpreting false theories helps teaches about our own world.


Figure 1: Hamiltonian Systems
Figure 2: Gauge Systems

phase space

\[ [x] \]

H

\[ x(1) \]

\[ x'(1) \]

\[ x''(1) \]